Let's make the world simpler
Last Last Time: Image filtering

\[ f[\ldots] \quad \quad \quad \quad \quad \quad \quad \quad \quad h[\ldots] \]

\[
h[m,n] = \sum_{k,l} f[k,l] g[m+k,n+l]
\]

Credit: S. Seitz
Filtering in spatial domain

\[
\begin{bmatrix}
1 & 0 & -1 \\
2 & 0 & -2 \\
1 & 0 & -1
\end{bmatrix}
\]

intensity image

\[
\ast
\]

=
Filtering in frequency domain
Phase and Magnitude

all natural images have about the same magnitude transform hence, phase seems to matter, but magnitude largely doesn't

\[ A = \pm \sqrt{R(u,v)^2 + I(u,v)^2} \]

\[ \phi = \tan^{-1} \frac{I(u,v)}{R(u,v)} \]

\[ F(u,v) = \text{SUM}\{ f(x,y) \exp(-j \times 2 \times \pi \times (u \times x + v \times y) / N) \} \]

\[ f(x,y) = \text{SUM}\{ F(u,v) \exp(j \times 2 \times \pi \times (u \times x + v \times y) / N) \} \]

where \( u = 0,1,2,...,N-1 \) and \( v = 0,1,2,...,N-1 \)
\( x = 0,1,2,...,N-1 \) and \( y = 0,1,2,...,N-1 \)
\( j = \sqrt{-1} \)
and SUM means double summation over proper \( x,y \) or \( u,v \) ranges

Euler's definition of \( e^{i\theta} \)

\[ e^{i\theta} = \cos \theta + i \sin \theta \]
Last Time: Inverse Ill Pose Problem

1 = ab

Hyperbola of feasible solutions
Last Time: Inverse III Pose Problem

Use $P(a, b \mid y = 1) = k P(y=1 \mid a, b) P(a, b)$

Likelihood function

$$P(y = 1 \mid a, b) = ke^{\frac{-(1-ab)^2}{2\sigma^2}}$$

Prior probability

$$P(a,b) = ke^{\frac{-(a-b)^2}{2\sigma^2}} \text{ if } a>0, b>0$$

$$= 0 \text{ otherwise}$$
Natural image statistics

Characteristic distribution with heavy tails

Histogram of image gradients

Dead leaves models
Introduced in the 60’s by Matheron (67) and popularized by Ruderman (97)

From Lee, Mumford and Huang 2001
Statistics of Images

Power spectra fall off as

$$|\hat{I}(v)| \sim \frac{1}{|v|^\alpha}$$

Natural Image Statistics

• Motivation: Separating into components
• Deal with ill pose: Bayesian Approach!
• Application 1: Denoising
• Application 2: Intrinsic Image
• Application 3: Reflection Separation
• Application 4: Deblurring
• Application 5: Digital Forensics
Denoising

Decomposition of a noisy image

\[ I_n(x, y) = n(x, y) + I(x, y) \]

White Gaussian noise: \( N(0, \sigma_n^2) \)
Natural image

Find \( I(x,y) \) that maximizes the posterior (maximum a posterioriy, MAP):

\[
\max_I p(I|I_n) = \max_I p(I_n|I) \times p(I) \\
= \max_I \exp\left(-\frac{1}{2} \frac{|I_n - I|^2}{\sigma_n^2}\right) \times \exp\left(-\frac{1}{2} I^T C^{-1} I\right)
\]
Goal: Intrinsic images
Again, we are trying to solve an ill-posed problem:

\[ 24 = ? \times ? \]
Retinex

too small? let it be zero!

From M. Tappen, PhD

(a) One column from the observed image.
(b) The derivative of the plot from (a).
(c) The estimate of the log shading

From M. Tappen, PhD

\[
\begin{bmatrix} 1 & -1 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & -1 \end{bmatrix}^T
\]
Separating reflections from a single image using local features

Anat Levin  Assaf Zomet  Yair Weiss

very simple cost function: it favors decompositions which have a small number of edges and corners. Surprisingly, this simple cost function gives the “right” decompositions for challenging real images.¹

Figure 1: (a) Original input image (constructed by summing the two images in b). (b) the correct decomposition. (c)-(g) alternative possible decompositions. Why should the decomposition in (b) be favored?

Figure 2: An input image and some decompositions
Three sources of information

1. Reconstruction constraint:

\[ \text{Estimated sharp image} \odot \text{Estimated blur kernel} = \text{Input blurry image} \]

2. Image prior:

Distribution of gradients

3. Blur prior:

Positive & Sparse
Image circulated on internet

Fonda Speaks To Vietnam Veterans At Anti-War Rally

Actress And Anti-War Activist Jane Fonda Speaks to a crowd of Vietnam Veterans as Activist and former Vietnam Vet John Kerry (LEFT) listens and prepares to speak next concerning the war in Vietnam (AP Photo)

http://www.cs.dartmouth.edu/farid/publications/significance06.pdf
Vision is hard!
The structure of ambient light
The structure of ambient light
The Plenoptic Function

The intensity $P$ can be parameterized as:

$$P (\theta, \phi, \lambda, t, X, Y, Z)$$

“The complete set of all convergence points constitutes the permanent possibilities of vision.” Gibson
Vision is hard because the world is complicated!

• Better Vision Algorithm
• Or, just to make the world simpler!
The summer vision project is an attempt to use our summer workers effectively in the construction of a significant part of a visual system. The particular task was chosen partly because it can be segmented into sub-problems which will allow individuals to work independently and yet participate in the construction of a system complex enough to be a real landmark in the development of "pattern recognition".
The “three weeks” vision project

The goal of PS2 is to solve computer vision in 3 weeks
A Simple Visual System

• A simple world
• A simple image formation model
• A simple goal
Today

- A simple world
- A catalog of edges
- Generic view assumption
- Constrained solver
- Edge detection: Canny
- Line detection
  - RANSAC
  - Hough transformation
A Simple World
A Simple World

MACHINE PERCEPTION OF THREE-DIMENSIONAL SOLIDS

by

LAWRENCE GILMAN ROBERTS

Submitted to the Department of Electrical Engineering on May 10, 1963, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

The problem of machine recognition of pictorial data has long been a challenging goal, but has seldom been attempted with anything more complex than alphabetic characters. Many people have felt that research on character recognition would be a first step, leading the way to a more general pattern recognition system. However, the multitudinous attempts at character recognition, including my own, have not led very far. The reason, I feel, is that the study of abstract, two-dimensional forms leads us away from, not toward, the techniques necessary for the recognition of three-dimensional objects. The per-

Complete Convex Polygons. The polygon selection procedure would select the numbered polygons as complete and convex. The number indicates the probable number of sides. A polygon is incomplete if one of its points is a collinear joint of another polygon.

A Simple World
A simple image formation model

Simple world rules:
• Surfaces can be horizontal or vertical.
• Objects will be resting on a white horizontal ground plane
A simple image formation model

Perspective projection

Parallel (orthographic) projection
A simple image formation model

World reference system

Camera plane

(right-handed reference system)
A simple image formation model

World coordinates

Image and projection of the world coordinate axes into the image plane

World coordinates

image coordinates

\[
x = X + x_0
\]

\[
y = \cos(\theta) Y - \sin(\theta) Z + y_0
\]
image coordinates

World coordinates

\[ x = X + x_0 \]
\[ y = Y \cos \theta - Z \sin \theta + y_0 \]
A simple goal

To recover the 3D structure of the world

We want to recover $X(x,y)$, $Y(x,y)$, $Z(x,y)$ using as input $I(x,y)$
Why is this hard?
Why is this hard?
Why is this hard?

Figure 1. (a) A line drawing provides information only about the x, y coordinates of points lying along the object contours. (b) The human visual system is usually able to reconstruct an object in three dimensions given only a single 2D projection (c) Any planar line-drawing is geometrically consistent with infinitely many 3D structures.
A simple visual system
The input image
• Proposition 1. The primary task of early vision is to deliver a small set of useful measurements about each observable location in the plenoptic function.

• Proposition 2. The elemental operations of early vision involve the measurement of local change along various directions within the plenoptic function.

Adelson, Bergen. 91

• Goal: to transform the image into other representations (rather than pixel values) that makes scene information more explicit.
Today

• A simple world
• A catalog of edges
• Generic view assumption
• Constrained solver
• Edge detection: Canny
• Line detection
  – RANSAC
  – Hough transformation
Object boundaries

1. Changes in surface orientation:
2. Occlusion boundaries: in this simple world, we will position the objects in such a way that objects do not occlude each other.
3. Contact edges: this is an edge between two objects but with no depth discontinuity.
4. Shadow edges: this can be harder than it seems. In this simple world, shadows are soft creating slow transitions between dark and light.
Edges

Occlusion

Change of Surface orientation

Contact edge

Horizontal 3D edge

Vertical 3D edge

Shadow boundary
Finding edges in the image

Image gradient:

\[ \nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) \]

Approximation image derivative:

\[ \frac{\partial I}{\partial x} \approx I(x, y) - I(x - 1, y) \]

Edge strength:

\[ E(x, y) = |\nabla I(x, y)| \]

Edge orientation:

\[ \theta(x, y) = \angle \nabla I = \arctan \frac{\partial I/\partial y}{\partial I/\partial x} \]

Edge normal:

\[ n = \frac{\nabla I}{|\nabla I|} \]
Finding edges in the image

\[
\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) \quad n = \frac{\nabla I}{|\nabla I|}
\]
Edge classification

• Figure/ground segmentation

• Occlusion edges
  – Occlusion edges are owned by the foreground

• Contact edges
Today

• A simple world
• A catalog of edges
• **Generic view assumption**
• Constrained solver
• Edge detection: Canny
• Line detection
  – RANSAC
  – Hough transformation
Generic view assumption: the observer should not assume that he has a special position in the world... The most generic interpretation is to see a vertical line as a vertical line in 3D.

Freeman, 93
Non-accidental properties

Perceptual Organization and Visual Recognition

David G. Lowe

Kluwer Academic Publishers

D. Lowe, 1985

<table>
<thead>
<tr>
<th>2-D Relation</th>
<th>3-D Inference</th>
<th>Examples</th>
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<tr>
<td>1. Collinearity of points or lines</td>
<td>Collinearity in 3-Space</td>
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<tr>
<td>2. Curvilinearity of points of arcs</td>
<td>Curvilinearity in 3-Space</td>
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<tr>
<td>3. Symmetry (Skew Symmetry?)</td>
<td>Symmetry in 3-Space</td>
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<td>4. Parallel Curves (Over Small Visual Angles)</td>
<td>Curves are parallel in 3-Space</td>
<td></td>
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<tr>
<td>5. Vertices—two or more terminations at a common point</td>
<td>Curves terminate at a common point in 3-Space</td>
<td></td>
</tr>
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</table>

Figure 4. Five nonaccidental relations. (From Figure 5.2, Perceptual organization and visual recognition [p. 77] by David Lowe. Unpublished doctoral dissertation, Stanford University. Adapted by permission.)

Biederman_RBC_1987
Non-accidental properties in the simple world

Using $\theta(x,y)$

Using $E(x,y)$

Using $\theta(x,y)$
Today

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  – Hough transformation
From edges to surface constraints

• Ground

\[ Y(x,y) = 0 \text{ if } (x,y) \text{ belongs to a ground pixel} \]

• Contact edge

\[ Y(x,y) = 0 \text{ if } (x,y) \text{ belongs to foreground and is a contact edge} \]

Next, things get a bit more complicated.
From edges to surface constraints

How can we relate the information in the pixels with 3D surfaces in the world?

- **Vertical edges**

Given the image, what can we say about X, Y and Z in the pixels that belong to a vertical edge?

\[
\begin{aligned}
Z &= \text{constant along the edge} \\
\partial Y / \partial y &= 1 / \cos(\theta)
\end{aligned}
\]
From edges to surface constraints

• Horizontal edges

Given the image, what can we say about \( X, Y \) and \( Z \) in the pixels that belong to an horizontal 3D edge?

\[
\begin{align*}
Y &= \text{constant along the edge} \\
\frac{\partial Y}{\partial t} &= 0 \\
\text{Where } t &= (-n_y, n_x) \\
\frac{\partial Y}{\partial t} &= \nabla Y \cdot t = -n_y \frac{\partial Y}{\partial x} + n_x \frac{\partial Y}{\partial y}
\end{align*}
\]
From edges to surface constraints

• What happens where there are no edges?

Assumption of planar faces:
  i.e. change of change is zero

\[
\frac{\partial^2 Y}{\partial x^2} = 0 \\
\frac{\partial^2 Y}{\partial y^2} = 0 \\
\frac{\partial^2 Y}{\partial y \partial x} = 0
\]

Information has to be propagated from the edges
A simple inference scheme

All the constraints are linear

\[ Y = 0 \] if \((x,y)\) belongs to a ground pixel

\[ \frac{\partial Y}{\partial y} = \frac{1}{\cos(\theta)} \] if \((x,y)\) belongs to a vertical edge

\[ \frac{\partial Y}{\partial t} = 0 \] if \((x,y)\) belongs to an horizontal edge

\[ \frac{\partial^2 Y}{\partial x^2} = 0 \]
\[ \frac{\partial^2 Y}{\partial y^2} = 0 \] if \((x,y)\) is not an edge

\[ \frac{\partial^2 Y}{\partial y \partial x} = 0 \]
Discrete approximation

We can transform every differential constraint into a discrete linear constraint on $Y(x,y)$.

$$\frac{dY}{dx} \approx Y(x,y) - Y(x-1,y)$$

A slightly better approximation (it is symmetric, and it averages horizontal derivatives over 3 vertical locations)
Discrete approximation

Transform the “image” $Y(x,y)$ into a column vector:

$$Y(x,y) - Y(x-1,y) = Y(2,2) - Y(1,2)$$

For $x=2$, $y=2$:

$$\frac{dY}{dx} \approx Y(x,y) - Y(x-1,y) = Y(2,2) - Y(1,2)$$
A simple inference scheme

A Y = b

Y = (A^T A)^{-1} A^T b

Matlab
Y = A \backslash b;
Results

Edge strength

3D orientation

Contact edges

Depth discontinuities

Edge normals
Changing view point

New view points:
Violations of simple world assumptions

Shading is due to painted stripes
Violations of simple world assumptions

Shading is due to illumination
Violations of simple world assumptions

Figure 24.9 The impossible steps. On the left, the horizontal stripes appear to be due to paint; on the right, they appear to be due to shading.
Impossible steps
Impossible steps
What is missing?

- Recognition
- The system can not do simple tasks like counting the number of cubes.
Different Approach

• A different approach to the one discussed here is model based scene interpretation where we could have a set of predefined models of the objects that can be present in the scene and the system should try to decide if they are present or not in the image, and recover their parameters (pose, color, etc.)

• Recognition allows indexing properties that are not directly available in the image. For instance, we can infer the shape of the invisible surfaces. Recognizing each geometric figure also implies extracting the parameters of each figure (pose, size, ...).
3D Cuboid Detector

Input image

Output detections

Synthesized New Views

http://vision.princeton.edu/projects/2012/SUNprimitive/
\[ S(I, p) = \sum_{i \in V} w_i^H \cdot \text{HOG}(I, p_i) + \sum_{i,j \in E} w_{ij}^D \cdot \text{Displacement}^{2D}(p_i, p_j) \]
\[ + \sum_{i,j \in E} w_{ij}^E \cdot \text{Edge}(I, p_i, p_j) + w^S \cdot \text{Shape}^{3D}(p) \]
Scoring Function

\[ S(I, p) = \sum_{i \in V} w_i^H \cdot \text{HOG}(I, p_i) + \sum_{ij \in \mathcal{E}} w_{ij}^D \cdot \text{Displacement}^{2D}(p_i, p_j) \]

\[ + \sum_{ij \in \mathcal{E}} w_{ij}^E \cdot \text{Edge}(I, p_i, p_j) + w^S \cdot \text{Shape}^{3D}(p) \]
Scoring Function

\[
S(I, p) = \sum_{i \in V} w_i^H \cdot \text{HOG}(I, p_i) + \sum_{ij \in E} w_{ij}^D \cdot \text{Displacement}^{2D}(p_i, p_j) + \\
\sum_{ij \in E} w_{ij}^E \cdot \text{Edge}(I, p_i, p_j) + \omega^S \cdot \text{Shape}^{3D}(p)
\]
Scoring Function

\[
S(I, p) = \sum_{i \in V} w_i^H \cdot \text{HOG}(I, p_i) + \sum_{ij \in \mathcal{E}} w_{ij}^D \cdot \text{Displacement}^{2D}(p_i, p_j) + \sum_{ij \in \mathcal{E}} w_{ij}^E \cdot \text{Edge}(I, p_i, p_j) + w^S \cdot \text{Shape}^{3D}(p)
\]
Scoring Function

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Scoring Function

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\[ + \sum_{ij \in \mathcal{E}} w_{ij}^E \cdot \text{Edge}(I, p_i, p_j) + w^S \cdot \text{Shape}^{3D}(p) \]
What objects are cuboids?

- cabinet (28/87)
- building (16/49)
- bed (15/22)
- night table (15/29)
- chest of drawers (10/10)
- box (9/18)
- desk (8/22)
- table (8/26)
- CPU (7/8)
- stand (7/11)
- brick (5/5)
- cabinets (5/22)
- kitchen island (5/6)
- night table occluded (5/12)
- stove (5/13)
- screen (5/16)

Others
- 97 categories
  (168/883)
Easy Cases
Detection Result
Multiple Instances
2D vs 3D

Ground truth
2D Detector
3D Detector
Cylinder Detection
Pyramid Detection
Today

- A simple world
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- Generic view assumption
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- Edge detection: Canny
- Line detection
  - RANSAC
  - Hough transformation
• Let’s look at gradients
Image Gradients

For 2D function \( f(x, y) \), the partial derivative is:

\[
\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon}
\]

For discrete data, we can approximate using finite differences:

\[
\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x + 1, y) - f(x, y)}{1}
\]
The gradient of an image:

\[ \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \]

The magnitude of the gradient:

\[ \|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \]

The direction of the gradient:

\[ \theta = \tan^{-1}\left(\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}\right) \]
Computing Image Gradients

This is a convolution with two simple filters:

\[
\frac{\partial f(x, y)}{\partial x} \quad \begin{bmatrix} -1 & 1 \end{bmatrix}
\]

\[
\frac{\partial f(x, y)}{\partial y} \quad \begin{bmatrix} -1 & 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & -1 \end{bmatrix}
\]
Computing Image Gradients

Other common gradient filters:

**Prewitt:**
\[ M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}; \quad M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \]

**Sobel:**
\[ M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}; \quad M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \]

**Roberts:**
\[ M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; \quad M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]
Computing Image Gradients

We usually limit high frequencies when computing gradient
Computing Image Gradients

We usually limit high frequencies when computing gradient
Computing Image Gradients

Useful fact #1: differentiation “commutes” with convolution

\[ \frac{df}{dx} \ast g = \frac{d}{dx} (f \ast g) = f \ast \frac{dg}{dx} \]

Useful fact #2: Gaussian is separable:

\[ G_2(x, y) = G_1(x)G_1(y) \]
Thus, combine smoothing with gradient computation:

\[
\nabla (f(x, y) * G_2(x, y)) = \begin{bmatrix}
    f(x, y) * (G'_1(x)G_1(y)) \\
    f(x, y) * (G_1(x)G'_1(y))
\end{bmatrix} = \begin{bmatrix}
    f(x, y) * G'_1(x) * G_1(y) \\
    f(x, y) * G_1(x) * G'_1(y)
\end{bmatrix}
\]
Computing Image Gradients

Can use different sigma to find gradients at different “scales”

1 pixel  3 pixels  7 pixels
Gradient Analysis

How are image gradients useful?
Edges

• An edge is a place of rapid change in the image intensity function.

- Image
- Intensity function (along horizontal scanline)
- First derivative

Edges correspond to extrema of derivative.
Edges

Silhouette Boundary

Convex Crease

Shadow Boundary

Concave Crease
Edges
Edges
Edge Detection

Useful for many applications in vision

- Segmentation
- Camera pose estimation
- 3D reconstruction
- Object classification
- Object recognition
- etc.
Canny Edge Detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression
4. Hysteresis thresholding
Canny Edge Detector

Original Image

Smoothed Gradient Magnitude
Nonmaximum suppression

- Eliminate all but local maxima in gradient magnitude (sqrt of sum of squares of x and y components)
- At each pixel p look along direction of gradient: if either neighbor is bigger, set p to zero
- In practice, quantize direction to horizontal, vertical, and two diagonals
- Result: “thinned edge image”
Canny Edge Detector

Smoothed gradient magnitude

Non-maximum suppression
Canny Edge Detector

Final stage: thresholding

Simplest: use a single threshold

Better: use two thresholds

- Find chains of touching edge pixels, all $\geq \tau_{low}$
- Each chain must contain at least one pixel $\geq \tau_{high}$
- Helps eliminate dropouts in chains, without being too susceptible to noise
- “Thresholding with hysteresis”
Canny Edge Detector

Non-maximum suppression

Canny edges
Canny Edge Detector

Original Image

Canny edges
Summary of Canny Edge Detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   • Thin wide “ridges” down to single pixel width
4. Hysteresis thresholding:
   • Define two thresholds: low and high
   • Use the high threshold to start edge curves and the low threshold to continue them
Today

- A simple world
- A catalog of edges
- Generic view assumption
- Constrained solver
- Edge detection: Canny
- Line detection
  - RANSAC
  - Hough transformation
Goal

Extract “structural features” from an image

• Non-accidental properties
Goal

What types of “structure” are in this image?
Goal

What types of “structure” are in this image?

- Straight lines
- Parallel lines
- Symmetric pairs of lines
- Trapezoids
- Monochromatic regions
- etc.
What types of “structure” are in this image?
Goal

What types of “structure” are in this image?

• Circles
• Ellipses
• Symmetries in color and texture
• etc.
Structure Detection

Algorithms for “structure detection”

- Line detection
- Circle detection
- Symmetry detection
Let’s first consider how to detect lines.
Let’s first consider how to detect **lines**
Desirable properties of a line detection algorithm?
Desirable properties of a line detection algorithm:

- Straight, long lines only
- Few missed or extra lines
- Provides confidence of prediction for each pixel
- Robust to differences in occlusion, noise, scale, rotation, translation, slight non-straightness, brightness, etc.
- Efficient computation
Line Detection

Not the same as edge detection:

- Edges are small-scale, local properties
- Lines are large-scale, structural properties
Line Detection

Applications:
• Removing radial distortion
• Camera pose estimation
• Segmentation
• Scene classification
• Object detection
• etc.
Line Detection

Applications:

- Removing radial distortion
- Camera pose estimation
- Segmentation
- Scene classification
- Object detection
- etc.
Please propose a line detection algorithm
Line Detection

OK, but what about this harder example?
Line Detection

This one?

Input → Output
Today

• A simple world
• A catalog of edges
• Generic view assumption
• Constrained solver
• Edge detection: Canny
• **Line detection**
  • **RANSAC**
  • Hough transformation
RANSAC in General

RANdom SAmples Consensus

Take many random samples of data

- Compute fit for each sample
- See how many points agree
- Remember the best
RANSAC for Line Detection

At beginning:
- Compute gradient direction $N$ and magnitude $G$

Iterate:
- Randomly choose a pixel $p$
- Choose a line $L$ through $p$
- Compute how well other pixels “support” $L$

At end:
- Report the line $L^*$ with the most “support”
RANSAC for Line Detection

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Point $p$ and Normal $N(p)$
RANSAC for Line Detection

At beginning:
• Compute gradient direction $N$ and magnitude $G$

Iterate:
• Randomly choose a pixel $p$
• Choose a line $L$ through $p$
• Compute how well other pixels “support” $L$

At end:
• Report the line $L^*$ with the most “support”
RANSAC for Line Detection

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Iterate:
• Randomly choose a pixel p
• Choose a line L through p
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\[
Support(L) = \sum_{q \in L} G(q) |N(p) \cdot N(q)|
\]
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Point \( p \) and Normal \( N(p) \)
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RANSAC for Line Detection

Many possible variants:

- **How to choose L?**
  - Point (and local gradient)
  - Point and angle
  - Two points
  - Three points
  - etc.

- **How compute “support” for L?**
  - \( \sum G(q) |N(p) \cdot N(q)| \)
  - Optimize L to fit “inliers”
  - etc.
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  - \[ \sum_{q \in L} G(q) |N(p) \cdot N(q)| \]
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RANSAC for Line Detection

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RANSAC for Line Detection

S = # samples (iterations)
T = time to evaluate each sample
n = width/height of image
d = degrees of freedom in line parameterization

Running time = O(ST) = O(n^d)
- S = O(n^{d-1})
- T = O(n)
RANSAC in General

RANdom SAmple Consensus

Take many random subsets of data

• Compute fit for each sample
• See how many points agree
• Remember the best

What else could this algorithm detect?
Detecting circles:

- Randomly choose three pixels p1, p2, and p3
- Compute a circle C through p1, p2, and p3
- Compute how well other pixels “support” C
Detecting circles:

- Randomly choose three pixels $p_1$, $p_2$, and $p_3$
- Compute a circle $C$ through $p_1$, $p_2$, and $p_3$
- Compute how well other pixels “support” $C$

What is the running time?
RANSAC for Circle Detection

Detecting circles:
- Randomly choose three pixels $p_1$, $p_2$, and $p_3$
- Compute a circle $C$ through $p_1$, $p_2$, and $p_3$
- Compute how well other pixels “support” $C$

How can we improve the running time?
RANSAC for Circle Detection

Possible parameterizations for circles:

- 6 dof: three points
- 5 dof: two points, one angle
- 4 dof: one point, one angle, one radius
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- 6 dof: three points
- 5 dof: two points, one angle
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![Diagram of circles with points and angles](image)
RANSAC for Circle Detection

Possible parameterizations for circles:
- 6 dof: three points
- 5 dof: two points, one angle
- 4 dof: one point, one angle, one radius
- 3 dof: one point, one radius
Today

• A simple world
• A catalog of edges
• Generic view assumption
• Constrained solver
• Edge detection: Canny
• Line detection
  • RANSAC
  • Hough transformation
Like RANSAC, except visit pixels p one-by-one and accumulate "support" (vote) for all primitives containing p in hash table bins
Hough Transform

At beginning:
  • Initialize all Hough space bins to zero

For each pixel sample p:
  • Add support to all Hough space bins representing primitives containing p

At end:
  • Find the Hough space bin(s) with the most support
Hough Transform for Line Detection

Example:

Image

Hough Space
Hough Transform for Line Detection

Example:

Image

Hough Space
Example:
Hough Transform for Line Detection

Example:

Image

Hough Space
Hough Transform for Line Detection

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Example:

Image

Hough Space
Hough Transform for Line Detection

Key question: how to parameterize Hough space?
Hough Transform for Line Detection

A 2 dof parameterization for lines: \( y = ax + b \)

Parameters: \( a \) and \( b \)

\[ a = \frac{dy}{dx} \]
Hough Transform for Line Detection

Every point in image lies on “line” of bins in Hough space with this parameterization.
Hough Transform for Line Detection

Problems with slope / intercept parameterization

- Non-uniform sampling of directions
- Can’t represent vertical lines
Hough Transform for Line Detection

Alternative: angle / distance parameterization

- Line represented as \((r, \theta)\) where
  - \(x \cos \theta + y \sin \theta = r\)
  - \(r = -\cos \theta \cdot x - \sin \theta \cdot y\)
  - \(N(L) = (\cos \theta, \sin \theta)\)
  - \(\text{dist}(L, p) = x \cos \theta + y \sin \theta - r\)
Hough Transform for Line Detection

Alternative: angle / distance parameterization

• Line represented as \((r, \theta)\) where
  
  \[
  x \cos \theta + y \sin \theta = r
  \]
  
  \[
  r = - \cos \theta \cdot x - \sin \theta \cdot y
  \]
  
  \[
  N(L) = (\cos \theta, \sin \theta)
  \]
  
  \[
  \text{dist}(L, p) = x \cos \theta + y \sin \theta - r
  \]
  
  \[
  L(r, \theta) \approx L(-r, \theta + \pi)
  \]
Hough Transform for Line Detection

Alternative: angle / distance parameterization

- Line represented as \((r, \theta)\)

  + Uniform sampling of angles

  -- Lines through point

    lie on sinusoid in \((r, \theta)\)
Hough Transform for Line Detection

Alternative: angle / distance parameterization

- Line represented as $(r, \theta)$
  
  + Uniform sampling of angles

  -- Lines through point
  
  lie on sinusoid in $(r, \theta)$
Hough Transform for Line Detection

Most people use angle / distance parameterization

- Line represented as \((r, \theta)\)
Hough Transform for Line Detection

Issue: How to select bucket size?
Hough Transform for Line Detection

Issue: How to select bucket size?

- Too small: poor performance on noisy data
- Too large: poor accuracy, possibility of false positives
Issue: How to select bucket size?

• Too small: poor performance on noisy data
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One solution:
• Large buckets + refinement
Hough Transform for Line Detection

Issue: How to select bucket size?
  • Too small: poor performance on noisy data
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One solution:
  • Large buckets + refinement

Least-squares minimization
Hough Transform for Line Detection

Issue: How to select bucket size?

- Too small: poor performance on noisy data
- Too large: poor accuracy, possibility of false positives

One solution:

- Large buckets + refinement
Hough Transform in General

What else can be detected with a Hough transform?

- Circles
- Ellipses
- Boxes
- Symmetries
- etc.

Anything that can be parameterized (in a small number of dimensions)
Hough Transform for Circle Detections

2D circles have 3 degrees of freedom
- Possible parameterization = 2D position and radius

So, each pixel gives rise to 2D sheet of values in 3D Hough space
Symmetry transform:
• Vote for midpoints between pixels
Hough Transform for Symmetry Detection

Symmetry transform:
  • Vote for midpoints between pixels
Hough Transform for Symmetry Detection

Symmetry transform:
  • Vote for midpoints between pixels
Hough Transform for Symmetry Detection

Symmetry transform:

- Vote for midpoints between pixels
- Weight votes by functions of distances, gradients, directions, etc.

\[ C(i, j) = D_\sigma(i, j) P(i, j) r_i r_j \]
Hough Transform for Symmetry Detection

Symmetry transform:
- Used for eye detection!!!
Hough Transform for Symmetry Detection

Reflective symmetry transform:

• Vote for bisector lines
Hough Transform and RANSAC

Very general computational techniques:

- Useful for detecting anything that can be parameterized in a low-dimensional space
Hough Transform vs. RANSAC?

How are algorithms similar / different?
Combine Edges and Lines

Detect edges locally
- Canny algorithm

Detect lines globally
- Hough algorithm

Combine

```plaintext
for each pixel p in the input image
    output_image(p) = \max \left( \frac{E(p)^\alpha \cdot H(L)^\beta \cdot (N(p) \cdot N(L))^\gamma}{L} \right)
end
```
Combine Edges and Lines

Input

Output

Canny edges

Strong Hough lines

Hough Transform
Summary

Problem:
- Structure detection

Focus: line detection
- RANSAC
- Hough transform

Extensions
- Circles and other primitives
- Symmetries

Applications
- Segmentation
- Alignment
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